Baylor University
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Department of Finance, Insurance & Real Estate

Risk Management
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Problem Set 7

Name: SOLUTIONS

Show your work and write as legibly as possible. Good luck!

Problem 1 (50 points)

Suppose the current value of a (non-dividend-paying) stock is $10,000, and the annual continuously compounded riskless rate of interest is 4%. Based on the example provided on pp. 9-14 from the "Derivatives Theory, Part 1" lecture note, solve parts A and B below.

A. (25 points)What is the "arbitrage-free" price for a forward contract on this stock which matures 1 year from today?

   SOLUTION: $F = Se^{rT} = \$10,000e^{.04} = \$10,408.11$

B. (25 points) Suppose the forward price is $10,400. Describe a profitable zero risk, zero net investment trading strategy involving the forward contract and its replicating portfolio. If you implement such a strategy, how much profit will you earn?

   SOLUTION: Since the forward price of $10,400 is below its "arbitrage-free" price by $8.11, it is undervalued. In order to take advantage of this mispricing in a way that involves zero risk and zero net investment, one should buy a 1-year forward contract for $10,400, sell the stock for $10,000, and lend $10,000 at the riskless rate. The following table succinctly summarizes this trading strategy:

Transaction	**Payoff now**	**Payoff @ *T***
Buy Forward	$0	S_T - $10,400
Sell Stock	$10,000	- S_T
Lend	($10,000)	$10,000$e^{.04}$ = $10,408.11
Arbitrage Profit	**$0**	**$8.11**

Problem 2 (50 points)

The price of a share of Zoom stock is currently $250. It is known that at the end of 1 year, the Zoom share price will be either $312.50 or $200. The riskless interest rate is 2% per year.

A. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of $250 by applying the replicating portfolio approach.

SOLUTION: According to the replicating portfolio approach:

$$\begin{aligned}
C_u &= Max(0, S_u - K) = 62.50 \\
C_d &= Max(0, S_d - K) = 0 \\
V_{RP} &= \Delta S + B \\
\Delta S &= \frac{C_u - C_d}{S(u-d)} S = \frac{62.50}{250(.45)} 250 = .5556(250) = 138.89 \\
B &= \frac{uC_d - dC_u}{e^{r\delta t}(u-d)} = \frac{1.25(0) - .8(62.50)}{e^{.02(1)}(.45)} = -108.91 \\
\therefore C = V_{RP} &= \Delta S + B = \$138.89 - \$108.91 = \$29.98.
\end{aligned}$$

B. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$250 by applying the risk neutral valuation approach.

SOLUTION: The risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.02} - .8}{1.25 - .8} =$ .4893. Since the stock is worth \$250(1.25) = \$312.50 at the u node and \$250(.8) = \$200 at the d node, this means that the call is only in the money at the u node; specifically, it is worth \$62.50 at that node. Therefore, the price of a one-year call option is

$$C = e^{-r\delta t}[qC_u + (1-q)C_d] = e^{-.02}[.4893(62.50)] = \$29.89.$$

C. (10 points) Calculate the price of a 1-year European put option on Zoom stock with an exercise price of \$250.

SOLUTION: According to the put-call parity equation, $C + Ke^{-r\delta t} = P + S$; therefore, $P = C + Ke^{-r\delta t} - S \Rightarrow P = 29.98 + 250e^{-.02(1)} - 250 = \$25.03$.

D. (20 points) Next, add another 1-year timestep; i.e., it is known that at the end of 2 years, the Zoom share price will be \$390.63, \$250, or \$160. Calculate the price of a 2-year European call option on Zoom stock with an exercise price of \$250. Also calculate the price of a 2-year European put option on Zoom stock with an exercise price of \$250.

SOLUTION: Note that the call option is only in the money at the uu node, where it is worth \$140.63. It is worthless at the ud and dd nodes. Applying risk neutral valuation, the price of a two-year call option is

$$\begin{aligned}
C &= e^{-2r\delta t}[q^2 C_{uu} + 2q(1-q)C_{ud} + (1-q)^2 C_{dd}] \\
&= e^{-.04}[.4893^2(140.63)] = \$32.35.
\end{aligned}$$

Regarding the (otherwise identical) 2-year put option, put-call parity indicates that the put is worth \$22.55:

$$P = C + Ke^{-2r\delta t} - S = 32.35 + \$250e^{-.04} - \$250 = \$22.55.$$