Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Risk Management Name: SOLUTIONS
Dr. Garven
Problem Set 8

Show your work and write as legibly as possible. Good luck!

Problem 1.

Suppose you are interested in determining arbitrage-free prices for a European call option and an (otherwise identical) European put option. The underlying stock does not pay dividends, and its current price is $S = \$18$. For both options, the exercise price $K = \$20$, $u = e^{\sigma\sqrt{\delta t}}$, $d = e^{-\sigma\sqrt{\delta t}}$, and the length of each timestep is $\delta t = 1/4$. Furthermore, the riskless rate of interest $r = 4\%$ per year, the underlying stock's volatility $\sigma = 25\%$ per year, and both options expire 1 year from today.

A. (30 points) What is the arbitrage-free price for the call option?

SOLUTION: Rather than calculate all possible node-specific prices for the underlying stock, it is simpler to determine the minimum number of up moves required in order for the call option to expire in the money. Let a = the smallest integer value $> \ln(K/Sd^n)/\ln(u/d)$. Since $\sigma = 25\%$ and $\delta t = 1/4$, $u = e^{\sigma\sqrt{\delta t}} = e^{.25\sqrt{1/4}} = 1.1331$ and $d = 1/u = .8825$, then $\ln(20/18(.8825^4))/\ln(1.1331/.8825) = 2.421$ and $a = 3$. This means that the call option will be in-the-money at nodes $uuud$ and $uuuu$, and from Pascal's Triangle, we know that there are 4 unique path sequences leading to the $uuud$ node and 1 unique path sequence leading to the $uuuu$ node.

The node $uuud$ stock price is $u^3dS = 1.1331^3(.8825)(18) = \$23.11$ and the node $uuuu$ stock price is $u^4S = 1.1331^4(18) = \$29.681$, so the call option payoffs at these nodes are $c_{uuud} = \$3.11$ and $c_{uuuu} = \$9.68$ respectively. Since the risk neutral probability of an up move is $q = \dfrac{e^{r\delta t} - d}{u - d} = \dfrac{e^{.04/4} - .8825}{1.1331 - .8825} = .5089$, it follows that

$$\begin{aligned} c &= e^{-.r4\partial t}\left[4q^3(1-q)c_{uuud} + q^4 c_{uuuu}\right] \\ &= e^{-.04}\left[4(.5089^3)(.4911)(\$3.11) + (.5089^4)(\$9.68)\right] = \$1.40. \end{aligned}$$

B. (20 points) What is the arbitrage-free price for the put option?

SOLUTION: Applying the put-call parity equation, we find that $c + Ke^{-rn\delta t} = p + S \Rightarrow p = \$1.40 + \$20e^{-.04} - 18 = \$2.61$.

Problem 2. For this problem, the following set of definitions applies:

C = current (European) call option price;
P = current (European) put option price;
S = current price of a non-dividend paying stock (underlying asset for both options);
K = exercise price (common to both options);
r = annualized riskless rate of interest;
T = time (in terms of number of years) to expiration; and
σ = annualized standard deviation of underlying asset's rate of return.

For each of the following scenarios (A through D), calculate the missing variable(s):

Scenario	***C***	***P***	***S***	***K***	***r***	σ	***T***
A	?	?	\$18	\$20	4%	25%	1.00
B	\$2.96	\$1.98	?	\$25	4%	25%	1.00
C	\$5.60	\$1.42	\$33	?	4%	25%	1.00
D	\$2.38	\$3.60	\$18	\$20	4%	?	1.00

SOLUTIONS:

A. SCENARIO A SOLUTION (20 points): Here, we must find the arbitrage-free prices for the call and put options. We do this by applying the Black-Scholes-Merton option pricing formula for pricing the call option. Once we have determined the arbitrage-free price for the call option, we can determine the corresponding arbitrage-free put option price by applying the put-call parity equation.

Since $c = SN(d_1) - Ke^{-rT}N(d_2)$, the key to calculating the value of the call option is in first calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of the underlying stock and the present value of the exercise price:

$$d_1 = \frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(18/20) + (.04 + .5(.25))1}{.25\sqrt{1}} = -0.1364.$$

Therefore, d_2= d_1 - $\sigma\sqrt{T}$= -0.1364 - .25$\sqrt{1}$ = -0.3864. Consequently, $N(d_1) = 44.57\%$, $N(d_2) = 34.96\%$, and the value of the call option is:

$$c = SN(d_1) - e^{-rT}KN(d_2) = 18(44.57\%) - e^{-.04(1)}(20)(34.96\%) = \$1.31.$$

According to the put-call parity equation, $c + Ke^{-rT} = p + S$; therefore, $p = \$1.31 + e^{-.04(1)}(\$20) - \$18 = \$2.52$.

B. SCENARIO B SOLUTION (10 points): Here, we must find the current market value of the stock. We do this by applying the put-call parity equation; since $c + Ke^{-rT} = p + S$, it follows that $S = c + Ke^{-rT} - p$ = 2.96 + $e^{-.04(1)}(25)$ - 1.98 = \$25.

C. SCENARIO C SOLUTION (10 points): Here, we must find the exercise price. We do this by applying the put-call parity equation; since $c + Ke^{-rT} = p + S$, it follows that $K = e^{rT}(p + S - c) = e^{.04(1)}(1.42 + \$33 - 5.60) = \$30$.

D. SCENARIO D SOLUTION (10 points): Here, we calculate the standard deviation, which requires computation by trial and error. Since 1) all parameter values for Scenarios A and D are the same except for option prices and volatility, and 2) Scenario D option prices are *higher* than Scenario A option prices, this implies that volatility must also be *higher* under Scenario D compared with Scenario A; i.e., greater than 25%. By either modeling this problem in Excel (e.g., consider using my Black-Scholes Calculator spreadsheet at http://fin4335.garven.com/spring2024/Black-ScholesCalculator.xlsx) or a web-based option pricing calculator such as the Black-Scholes Calculator at https://goodcalculators.com/black-scholes-calculator/, a volatility measure of 40% will produce the call and put prices listed in the table (i.e., \$2.38 and \$3.60 respectively).