Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Risk Management
Dr. Garven
Problem Set 9

Name: SOLUTIONS

Show your work and write as legibly as possible. Good luck!

Consider two firms which differ from each other in terms of value and risk of corporate assets, and degree of financial leverage. Firm 1 owns assets worth \$30,000,000, and has issued zero coupon bonds with a face value of \$15,000,000. On the other hand, Firm 2 owns assets worth \$50,000,000, and has issued zero coupon bonds with a face value of \$30,000,000. The standard deviation for Firm 1's assets is $\sigma = 30\%$, whereas the standard deviation for Firm 2's assets is $\sigma = 40\%$. Assume that both firms will be liquidated one year from today and that the rate of interest is 3%.

1. What is the fair market value for the bonds issued by Firm 1? What is the dollar value of Firm 1's limited liability put option? What is the yield to maturity, credit risk premium, and the risk neutral probability of default for Firm 1's bonds?

   SOLUTION: We start by calculating the fair market value for riskless bonds, which is

   $$V(B) = Be^{-rT} = \$15,000,000e^{-.03} = \$14,556,683.$$

   Next, we calculate the dollar value of Firm 1's limited liability put option, the formula for which is

   $$V(Max(B-F,0) = e^{-rT}BN(-d_2) - V(F)N(-d_1).$$

   We start by calculating $-d_1$ and $-d_2$:

   $$-d_1 = -\frac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = -\frac{\ln(\$30/\$15) + (.03 + .5(.09))}{.30} = -2.5605, \text{ and}$$
   $$-d_2 = -d_1 + \sigma\sqrt{T} = -2.2605.$$

   Thus, the value of the limited liability put option is

   $$V(Max(B-F,0) = \$14,556,683(.0119) - \$30,000,000(.0052) = \$16,370.97,$$

   and the fair market value for the risky bonds issued by Firm 1 is

   $$V(D) = V(B) - V(Max(B-F,0) = \$14,556,683 - \$16,370.97 = \$14,540,312.$$

   The yield to maturity is the rate of return that will cause the risky bond value of \$14,540,312 to grow to a maturity value of \$15 million; i.e.,

$$\$15,000,000 = \$14,540,312e^{YTM(T)};$$
$$\$15,000,000/\$14,540,312 = e^{YTM(1)};$$
$$YTM = \ln(\$15,000,000/\$14,540,312) = 3.11\%.$$

The credit risk premium for Firm 1 is $YTM - r = 0.11\%$; i.e., 11 basis points, and finally, the risk neutral probability of default for Firm 1's bonds is $N(-d_2) = N(-2.2605) = 1.19\%$. Since the probability of default is quite small, the credit risk premium demanded by investors is (not surprisingly) also quite small.

2. What is the fair market value for the bonds issued by Firm 2? What is the dollar value of Firm 2's limited liability put option? What is the yield to maturity, credit risk premium, and probability of default for Firm 2's bonds?

SOLUTION: Next we undertake the same calculations for Firm 2 as we did for Firm 1. The fair market value for Firm 2's bonds is

$$V(B) = Be^{-rT} = \$30,000,000e^{-.03} = \$29,113,366.$$

Next, we calculate the dollar value of Firm 1's limited liability put option; we begin by first calculating $-d_1$ and $-d_2$:

$$-d_1 = -\frac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = -\frac{\ln(\$50/\$30) + (.03 + .5(.16))}{.40} = -1.5521, \text{ and}$$
$$-d_2 = -d_1 + \sigma\sqrt{T} = -1.1521.$$

Thus, the value of the limited liability put option is

$$V(Max(B - F, 0) = \$29,113,366(.1246) - \$50,000,000(.0603) = \$612,732,$$

and the fair market value for the risky bonds issued by Firm 1 is

$$V(D) = V(B) - V(Max(B - F, 0) = \$29,113,366 - \$612,732 = \$28,500,634.$$

The yield to maturity is the rate of return that will cause the risky bond value of \$28,500,634 to grow to a maturity value of \$30 million; i.e.,

$$\$30,000,000 = \$28,500,634e^{YTM(T)};$$
$$\$30,000,000/\$28,500,634 = e^{YTM(1)};$$
$$YTM = \ln(\$30,000,000/\$28,500,634) = 5.13\%.$$

The credit risk premium for Firm 1 is $YTM - r = 2.13\%$; i.e., 213 basis points, and finally, the risk neutral probability of default for Firm 1's bonds is $N(-d_2) = N(-1.1521) = 12.46\%$. Since the probability of default for Firm 2 is much higher than it is for Firm 1, the credit risk premium demanded by investors is (not surprisingly), also much higher.

3. Suppose an insurer offers credit enhancement schemes to both firms which ensure that neither firm will default on its debt. Assuming that the credit enhancement market is competitively structured, how much should each firm expect to pay for this service, and what impact will credit enhancement have on the yields to maturity for these bonds?

   SOLUTION: The fair premiums are equal to the values of the limited liability put option for the two firms; i.e., for Firm 1 the fair premium is \$16,370.97, and for Firm 2 the fair premium is \$612,732.25. By making the bonds of both firms riskless from the viewpoint of investors, the credit enhancement will cause the probability of default for these bonds to go to zero, and the yields to maturity for these bonds will be equal to the riskless rate of interest (3%). Since investors do not bear any credit risk, the credit risk premium required by investors must also be zero.