BAYLOR UNIVERSITY
HANKAMER SCHOOL OF BUSINESS
DEPARTMENT OF FINANCE, INSURANCE & REAL ESTATE

Risk Management
Dr. Garven
Sample Midterm Exam #1

Name SOLUTIONS

Notes:

1. Please read these instructions carefully.

2. This test consists of 2 problems worth 32 points each and a 3rd problem worth 36 points; thus, the maximum number of points possible for this exam is 100.

3. If you need extra space for solving any of the problems on this exam, use the extra pages provided.

4. You may have the entire class period to complete this examination. Be sure to show your work as well as provide a complete answer for each problem; i.e., besides producing numerical results, also explain your results in plain English.

Good luck!

Problem #1 (32 points)

Consider the following four lotteries, A through D. Note that the rows indicate the state-contingent probabilities ($f(W_s)$) associated with receiving the cash payments indicated at the top of each column. For example, lottery A provides a 1/3 chance at $1,000, a 1/3 chance at $2,000, and a 1/3 chance at $3,000, whereas lottery B provides a 3/10 chance at $1,000, a 1/2 chance at $2,000, and a 1/5 chance at $3,000, and so forth:

	Cash Payments		
Lottery	**$1,000**	**$2,000**	**$3,000**
A	1/3	1/3	1/3
B	.3	.5	.2
C	.12	.7	.18
D	.5	.3	.2

A. (8 points) If given a choice between lottery A or lottery D, which of these two lotteries would an arbitrarily risk averse decision-maker select? Explain why.

SOLUTION: Expected utility is higher for one risk compared with another if there is stochastic dominance. Here, we compare lotteries A and D to determine whether one of these lotteries stochastically dominates the other.

Lottery	$F(W_{A,s})$	$F(W_{D,s})$
$1,000	1/3	.5
$2,000	2/3	.8
$3,000	1	1

Since $F(W_{D,s}) \geq F(W_{A,s})$ for all s, this implies that A FOSD D, which implies the expected utility of lottery A must exceed that of lottery D for an arbitrarily risk averse decision-maker.

B. (8 points) If given a choice between lottery B or lottery C, which of these two lotteries would an arbitrarily risk averse person select? Explain why.

Lottery	$F(W_{B,s})$	$F(W_{C,s})$
$1,000	.3	.12
$2,000	.8	.82
$3,000	1	1

As the table above indicates, FOSD does *not* obtain. Next, we check for SOSD:

W_s	$F(W_{B,s})$	$F(W_{C,s})$	$F(W_{B,s})-F(W_{C,s})$
\$1,000	.3	.12	.18
\$2,000	.8	.82	-.02
\$3,000	1	1	0
		$\sum_{s=1}^{n}(F(W_{B,s})-F(W_{C,s}))$	.16

As the table above indicates, lottery *C* SOSD lottery *B*, which implies the expected utility of lottery *C* must exceed that of lottery *B* for an arbitrarily risk averse decision-maker.

C. (8 points) If given a choice between lottery *A* or lottery *C*, which of these two lotteries would an arbitrarily risk averse person select? Explain why.

<u>SOLUTION</u>: Here, we compare lotteries *A* and *C* to determine whether one of these lotteries stochastically dominates the other.

Lottery	$F(W_{A,s})$	$F(W_{C,s})$
\$1,000	1/3	.12
\$2,000	2/3	.82
\$3,000	1	1

As the table above indicates, FOSD does *not* obtain. Next, we check for SOSD:

W_s	$F(W_{A,s})$	$F(W_{C,s})$	$F(W_{A,s})-F(W_{C,s})$
\$1,000	1/3	.12	.2133
\$2,000	2/3	.82	-15.33
\$3,000	1	1	0
		$\sum_{s=1}^{n}(F(W_{A,s})-F(W_{C,s}))$	.06

As table above indicates, lottery *C* SOSD lottery *A*, which implies the expected utility of lottery *C* must exceed that of lottery *A* for an arbitrarily risk averse decision-maker.

D. (8 points) Which of these four lotteries is most preferred by an arbitrarily risk averse decision-maker? Explain why.

<u>SOLUTION</u>: Since lottery *C* stochastically dominates lotteries *A* and *B* and lottery *A*

stochastically dominates lottery *D*, this implies that the expected utility of lottery *C* must exceed expected utilities for lotteries *A*, *B*, and *D* for all risk averse utilities.

Although such a numerical illustration is not necessary, consider expected utility calculations for the square root utility function $U(W) = \sqrt{W}$:

Lottery	A	B	C	D
$E(U(W))$	43.70	42.80	44.96	40.18

Thus, $E(U(W_C)) > E(U(W_A)) > E(U(W_B)) > E(U(W_D))$. Furthermore, one can perform expected utility calculations for virtually any risk averse utility, and the same order sequence obtains. Although this question did not require checking for stochastic dominance between lotteries *A* and *B*, it turns out that lottery *A* SOSD *B*, so it is to be expected that $E(U(W_A)) > E(U(W_B))$.

Problem #2 (32 points)

Rocketman has a parabolic utility function; i.e., $U(W) = W^2$ and initial wealth $W_0 = \$0$. He is offered a lottery involving a fair coin toss; i.e., $p_{heads} = p_{tails} = 50\%$. If heads come up, then Rocketman receives \$64, but if tails come up, he receives \$9.

A. (8 points) Assume that Rocketman decides to take this risk. Compute the following: 1) expected value of wealth, 2) expected utility of wealth, 3) utility of the expected value of wealth, and 4) the certainty equivalent of wealth.

SOLUTION:

$$E(W) = \sum_{s=1}^{n} p_s W_s = .5(\$64) + .5(\$9) = \$36.50;$$

$$E(U(W)) = \sum_{s=1}^{n} p_s U(W_s) = .5(4,096) + .5(81) = 2,088.5;$$

$$U(E(W)) = U(36.5) = 36.5^2 = 1,332.25; \text{ and}$$

since $2,088.5 = E(U(W)) = U(W_{CE}) \Rightarrow W_{CE} = \sqrt{2,088.5} = \$45.70.$

B. (8 points) Compare the expected utility of wealth with the utility of the expected value of wealth and explain *why* there is a difference between these two values.

SOLUTION: Since the utility of the gamble $E(U(W)) = 2,088.5$ whereas the utility of the sure bet is $U(E(W)) = 1,332.25$, this means that Rocketman is risk loving. When given a choice between $36.50 for certain compared with a gamble which features the same expected value ($36.50) but also adds risk, Rocketman prefers the gamble.

C. (8 points) Compare the expected value of wealth with its certainty equivalent and explain *why* there is a difference between these two values.

SOLUTION: The difference between $E(W) = \$36.50$ and $W_{CE} = \$45.70$ corresponds to the risk premium; i.e., the risk premium is $\lambda = E(W) - W_{CE} = \$36.50 - \$45.70 = -\$9.20$. Since Rocketman is risk loving, this implies that he is willing to pay an unfair (greater than expected value) price for the gamble; thus, the negative risk premium.

D. (8 points) At what price is which Rocketman is indifferent between gambling and not gambling?

SOLUTION: Since $E(U(W)) = U(W_{CE}) \Rightarrow W_{CE} = \$45.70,$ this is the price at which Rocketman is indifferent. In other words, he regards receiving $45.70 with certainty as being equivalent in a utility sense to the lottery described in this problem.

Problem #3 (36 points)

Neo has the following utility function: $U(W) = \ln W$. His initial wealth is $100 and he is offered a coin toss which pays $60 (with probability .5) if the coin comes up heads and -$60 (also with probability .5) if the coin comes up tails.

A. (12 points) What is Neo's certainty equivalent of wealth and risk premium?

SOLUTION: To determine Neo's certainty equivalent of wealth (W_{CE}), we must calculate expected utility, set expected utility equal to the utility of the certainty equivalent of wealth, and then solve for the certainty equivalent of wealth. Once the certainty equivalent of wealth has been determined, then we can find Neo's risk premium (λ) by subtracting the certainty equivalent of wealth from expected wealth.

$$E(W) = \sum_{s=1}^{n} p_s W_s = .5(\$160) + .5(\$40) = \$100;$$

$$E(U(W)) = \sum_{s=1}^{n} p_s U(W_s) = .5(5.0752) + .5(3.6889) = 4.382;$$

thus, since $4.382 = E(U(W)) = U(W_{CE}) \Rightarrow W_{CE} = e^{4.382} = \$80 \Rightarrow \lambda = E(W) - W_{CE} = \$20.$

B. (12 points) Suppose that Morpheus is offered this same gamble. Like Neo, Morpheus has $W_0 = \$100$. However, Morpheus's utility $U(W) = W^{.5}$. What is Morpheus's certainty equivalent of wealth and risk premium?

SOLUTION:

$$E(W) = \sum_{s=1}^{n} p_s W_s = .5(\$160) + .5(\$40) = \$100;$$

$$E(U(W)) = \sum_{s=1}^{n} p_s U(W_s) = .5(12.6491) + .5(6.3246) = 9.4863;$$

thus, since $9.4863 = E(U(W)) = U(W_{CE}) \Rightarrow W_{CE} = 9.4863^2 = \$90 \Rightarrow \lambda = E(W) - W_{CE} = \$10.$

C. (12 points) Who is more risk averse, Neo or Morpheus? Explain why.

SOLUTION: Neo is more risk averse than Morpheus. This is implied by the fact that Neo's $\lambda = \$20$ whereas Morpheus's $\lambda = \$10$. Furthermore, Morpheus's $R_A(W) = .5/W$, which indicates that he has a lower degree of risk aversion than Neo.