Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Risk Management
Dr. Garven
Sample Midterm Exam 2

Name: SOLUTIONS

Notes:

1. This exam consists of four problems worth 32 points each. Only three of the four problems need to be completed; for students who complete all four problems, I will count the three highest-scoring problems toward the exam grade.

2. The maximum number of points possible on this exam is 100, consisting of 96 possible points on the problems plus 4 points for legibly writing your name on the cover page in the space provided.

3. If you need extra space for solving any of the problems on this exam, use the extra pages (labeled as "Extra Space for solving Problem #n") that are included with this exam booklet.

4. You may have the entire class period to complete this examination. Be sure to show your work and provide a complete answer for each problem; i.e., in addition to producing numerical results, explain your results in plain English.

Good luck!

Problem 1 (32 points)

An entrepreneur has initial wealth of \$88. Her initial wealth is invested in two buildings, each of which is worth \$40. Her remaining \$8 in initial wealth is invested in cash. Each building has a 25% chance of being destroyed and a 75% chance of not suffering any damage. Because the buildings are located far away from each other, these risks are statistically independent.

Since the entrepreneur has \$8 in cash, she can use some or all of this money to purchase actuarially fair insurance policies to cover her risks. Note that the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy.

1. (8 points) Given the entrepreneur's cash resources, if she covers 60% of the first building's potential loss, what is the maximum level of coverage (in terms of proportion of potential loss) that she can purchase against the risk that the second building will be destroyed?

   SOLUTION: The expected value of each building's potential loss is $E(L) = \sum_{s=1}^{n} p_s L_s = .25(40) = \$10$. Since the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy, this means that the premium for the first building's insurance policy is $.6 \sum_{s=1}^{n} p_s L_s = (.6).25(40) = \$6$. Since the entrepreneur has \$8 in cash, the maximum she can pay to insure the second building is \$2, which implies a maximum level of coverage of 20%.

2. (8 points) Given the entrepreneur's cash resources, what is the maximum level of coverage (in terms of proportion of potential loss) for each building that will result in the same premium being paid for each policy?

   SOLUTION: Since the most that the entrepreneur can spend on insurance is \$8 and the expected value of each building's potential loss is \$10, this means that she can cover 40% of the potential loss for each building for a premium of \$4 per building.

3. (8 points) Suppose the entrepreneur's utility function is $U(W) = \sqrt{W}$. Show that the entrepreneur is better off if she insures both buildings at the same level of coverage (for a total premium of \$8) than she would be if she implemented the risk management strategy implied in Part A of this problem.

   SOLUTION: Since the risks are statistically independent, this implies that the joint probability distribution for both risks consists of 4 possible states of the world: 1) no loss on either building (with probability .75 x .75 = 56.25%), 2) losses on both buildings (with probability .25 x .25 = 6.25%), 3) loss on building 1 and no loss on building 2 (with probability .75 x .25 = 18.75%) and 4) no loss on building 1 and loss on building 2 (with probability .25 x .75 = 18.75%). Furthermore, we need to derive an equation for state-contingent wealth in each of these states. Under this scenario (where she covers 40% of the potential loss for each building for a premium of \$4 per building), this implies that state-contingent wealth is $W_s = W_0 - \alpha_1 p_1 - \alpha_2 p_2 - (1-\alpha_1)L_{1s} - (1-\alpha_2)L_{2s} = 88 - 8 - .6L_{1s} - .6L_{2s}$. Thus, the following distribution of state-contingent wealth is implied by this equation:

State	p_s	L_{1s}	L_{2s}	W_s	$U(W_s)$
no loss on either building	56.25%	0	0	80	8.9443
losses on both buildings	6.25%	40	40	32	5.6569
loss on building 1 and no loss on building 2	18.75%	40	0	56	7.4833
no loss on building 1 and loss on building 2	18.75%	0	40	56	7.4833
	Expected Value:	**10**	**10**	**68**	**8.1909**

Thus, the expected utility of this risk management decision is 8.1909. Now suppose that the entrepreneur implements the risk management decision implied in part A. In other words, she covers 60% of the first building's potential loss for a premium of \$6 and 20% of the second building's potential loss for a premium of \$2. This risk management decision results in the following distribution of state-contingent wealth:

State	p_s	L_{1s}	L_{2s}	W_s	$U(W_s)$
no loss on either building	56.25%	0	0	80	8.9443
losses on both buildings	6.25%	40	40	32	5.6569
loss on building 1 and no loss on building 2	18.75%	40	0	64	8.0000
no loss on building 1 and loss on building 2	18.75%	0	40	48	6.9282
	Expected Value:	**10**	**10**	**68**	**8.1837**

Thus the expected utility of this alternative risk management decision is 8.1837, which implies that she is better off if she insures both buildings at the same level of coverage.

4. (8 points) Explain *why* the expected utility of having the same level of coverage on both buildings is higher than the expected utility of having different levels of coverage.

   SOLUTION: This raises an interesting question - why is the expected utility of having the same level of coverage on both buildings higher than the expected utility of having different levels of coverage? The answer is quite simple. We know from the expected utility theory that a mean preserving spread will always produce a lower expected utility ranking. In this problem, the alternative risk management decision involving different levels of coverage represents a mean preserving spread of the risk management decision involving the same level of coverage on both buildings. Looking closer, the source of the greater dispersion associated with the alternative risk management decision occurs whenever one building is destroyed and the other building doesn't suffer any damage. Comparing these tables, state-contingent wealth in the 3^{rd} and 4^{th} states varies when there are different levels of coverage, but is the same when the same level of coverage is selected for both buildings.

Problem 2 (32 points)

Consider a society with three kinds of drivers: safe, dangerous, and crazy. There is an equal number of each of these drivers, and each driver has initial wealth of \$150 and utility $U(W) = \sqrt{W}$. In any given year, a safe driver gets into an accident with probability $p_s = 0.1$; a dangerous driver gets into an accident with probability $p_d = 0.2$, and a crazy driver gets into an accident with probability $p_c = 0.3$. An accident leads to repair costs of \$50, and has no other consequences.

Suppose that an insurance company offers full coverage insurance policies to these drivers; i.e., when accidents occur, claim payments of \$50 are made which fully cover repair costs. The drivers decide whether to purchase these full coverage policies.

A. (8 points) Suppose the insurance company can distinguish the three types of drivers, and offers each driver an actuarially fair full coverage insurance policy. Which of the drivers will buy such a policy?

SOLUTION: If insurance is actuarially fair, we know from the Bernoulli hypothesis that risk averse policyholders will prefer full coverage.

Simply invoking the Bernoulli hypothesis is sufficient for earning full credit. However, it is also acceptable if the student demonstrates that the safe, dangerous, and crazy drivers all have higher expected utility when insurance is actuarially fair. Actuarially fair insurance for the safe drivers costs $E(L_s) = \sum_{s=1}^{n} p_{s,s}L_s = .1(50) = \$5$. For dangerous drivers, actuarially fair insurance costs $E(L_d) = \sum_{s=1}^{n} p_{s,d}L_s = .2(50) = \$10$, and for crazy drivers, actuarially fair insurance costs $E(L_c) = \sum_{s=1}^{n} p_{s,c}L_s = .3(50) = \$15$.

Next, we compute expected utility when there is no insurance:

- EU (uninsured safe driver): $E(U(W_s)) = \sum_{s=1}^{n} p_{s,s}U(W_s) = .1\sqrt{100} + .9\sqrt{150} = 12.0227$.
- EU (uninsured dangerous driver): $E(U(W_d)) = \sum_{s=1}^{n} p_{s,d}U(W_s) = .2\sqrt{100} + .8\sqrt{150} = 11.7980$.
- EU (uninsured crazy driver): $E(U(W_c)) = \sum_{s=1}^{n} p_{s,c}U(W_s) = .3\sqrt{100} + .7\sqrt{150} = 11.5732$.

Expected utility is always higher when these drivers can fully insure at actuarially fair prices:

- EU (insured safe driver): $E(U(W_s)) = \sqrt{145} = 12.0416$.
- EU (insured dangerous driver): $E(U(W_d)) = \sqrt{140} = 11.8322$.
- EU (insured crazy driver): $E(U(W_c)) = \sqrt{135} = 11.6190$.

B. (8 points) Now suppose that the insurance company cannot distinguish the three types of drivers. Therefore, it decides to offer to offer a full coverage insurance policy to all drivers for a price of \$10. Show that safe drivers would not be willing to purchase such a policy, whereas dangerous and crazy drivers would purchase such a policy.

SOLUTION: If a full coverage policy costs \$10, this means that the expected utility of full coverage is $\sqrt{140} = 11.8322$ for everyone. Since safe drivers have higher expected utility when they remain uninsured, they will not be willing to purchase this policy. We have already determined that the dangerous drivers will insure at a price of \$10, so we know that they will stay in the market. The crazy drivers are certainly quite happy about this deal; they would have insured at a price of \$15 but are now getting an even better deal. However, now the insurer will lose money because the crazy drivers have an expected cost of \$15 and the insurer is only collecting \$10 in premium from each of these drivers.

C. (8 points) Since safe drivers are not willing to purchase the policy described in part B, the insurance company decides to increase the price of the policy so that it won't lose money. This time, it offers a full coverage insurance policy for a price of \$12.50. Show that safe and dangerous drivers would not be willing to purchase such a policy, whereas crazy drivers would purchase such a policy.

SOLUTION: If a full coverage policy costs \$12.50, this means that the expected utility of full coverage is $\sqrt{137.50} = 11.7260$ for the dangerous and crazy drivers. Since dangerous drivers have higher expected utility when they remain uninsured, they will not be willing to purchase this policy. We have already determined that the dangerous drivers will insure at a price of \$10, so we know that they will stay in the market. However, now the insurer will lose money because the crazy drivers have an expected cost of \$15 and the insurer is only collecting \$12.50 in premium from each of these drivers.

D. (8 points) This problem numerically illustrates how markets can fail due to adverse selection; i.e., even though safe and dangerous drivers are risk averse and would like to purchase fairly priced insurance, the market will only provide fairly priced insurance for the crazy drivers. Explain the intuition for why this occurs.

SOLUTION: The basic dilemma here is that when the insurer tries to charge an average premium, then the low risk insureds drop out of the risk pool, which in turn raises the average cost of providing insurance for the remaining policyholders. The insurer gets stuck in a vicious cycle; when she raises premiums in response to the lower risk policyholders dropping out, this in turn aggravates the problem even more. This process continues until only the worst risks are left.

Problem 3 (32 points)

The following table lists the state-contingent returns on Security A ($r_{A,s}$) and Security B ($r_{B,s}$):

State of Economy	p_s	$r_{A,s}$	$r_{B,s}$
Bust	50%	-0.20	+0.25
Boom	50%	+0.40	-0.05

A. (8 points) What are the expected returns for Security A and Security B?

SOLUTION: $E(r_A) = \sum_{s=1}^{n} p_s r_{A,s}$= .5(-.2) + .5(.4) = .10; and

$E(r_B) = \sum_{s=1}^{n} p_s r_{B,s}$= .5(.25) + .5(-.05) = .10.

B. (8 points) What are the standard deviations of the returns for Security A and Security B?

SOLUTION:
$\sigma^2_{r_A} = \sum_{s=1}^{n} p_s (r_{A,s} - E(r_A))^2 = .5(-.2 - .10)^2 + .5(.4 - .10)^2 = .09$; therefore σ_{r_A}= .30; and

$\sigma^2_{r_B} = \sum_{s=1}^{n} p_s (r_{B,s} - E(r_B))^2 = .5(.25 - .10)^2 + .5(-.05 - .10)^2 = .02$; therefore σ_{r_B}= .150.

C. (8 points) Find the expected return and standard deviation for least possible risky combination of Security A and Security B. What is the composition of this portfolio (i.e., find the security weights w_A and w_B)?

SOLUTION:
$\sigma_{AB} = \sum_{s=1}^{n} p_s (r_{As} - E(r_A))(r_{Bs} - E(r_B))$ = .5(-.2 -.10)(.25 -.10) + .5(.40 -.10)(-.05-.10) = -.045.

Therefore, $w_A = \dfrac{\sigma^2_B - \sigma_{AB}}{\sigma^2_A + \sigma^2_B - 2\sigma_{AB}} = \dfrac{.0225 + .045}{.09 + .0225 + .09}$= 1/3. Consequently, $w_B = 1 - w_A$= 2/3, and r_{mvp} = .10. Furthermore, since ρ_{AB} = -.045/(.30)(.15) = -1, the standard deviation of the least possible risky combination of security A and security B is zero.

D. (8 points) Suppose your initial wealth is $1,000 and that you can borrow or lend up to $1,000 at the riskless rate of interest of 3% during the course of the next year. Given this information, describe the most profitable *riskless* trading strategy which can be implemented, and calculate the profit from implementing this strategy.

SOLUTION: The most obvious riskless investment strategy would involve investing your initial wealth of $1,000 in a riskless bond which yields 3%. Furthermore, since you can borrow up to $1,000, you could also lever this strategy by investing $2,000 at 3% and then paying back the principal plus interest on the \$1,000 loan. However, since the opportunity cost of capital for a riskless investment *is* the riskless rate of interest, these strategies do not increase your net worth; i.e., their net present values are \$0.

It is possible to increase your net worth without taking any risk by investing your initial wealth of \$1,000 plus an additional \$1,000 of borrowed money in the minimum variance portfolio; the value of such an investment after 1 year is $\$2,000e^{.10} - 1,000e^{.03}$= \$2,210.34 – \$1,072.51 = \$1,137.83, which implies an expected return totaling 13.78%. The net present value of this riskless arbitrage strategy is $NPV = \$1,137.83e^{-.07}$ - \$1,000 = \$60.91.

Problem 4 (32 points)

Suppose you have two stocks in your portfolio, *Max* and *Min*. The expected return of *Max* is 20% and the expected return of *Min* is 12%. The standard deviation of *Max* is 42% and the standard deviation of *Min* is 28%. The correlation between the two securities is zero. Suppose the riskless asset has an expected return of 4%.

A. What is the mean and standard deviation of the minimum variance portfolio combination of *Max* and *Min*?

SOLUTION: The ratio given by $w_{\text{Min}} = \dfrac{\sigma^2_{Max} - \sigma_{Max,Min}}{\sigma^2_{Min} + \sigma^2_{Max} - 2\sigma_{Max,Min}}$ provides a value for w_{Min} which minimizes portfolio variance; therefore, $w_{\text{Min}} = \dfrac{.1764}{.0784 + .1764} = .69$; therefore, $E(r_p) = .31 \times E(r_{Max}) + .69 \times E(r_{Min}) = .31\,(20\%) + .69\,(12\%) = 14.46\%$, and $\sigma_p = \sqrt{.31^2 \times \sigma^2_{Max} + .69^2 \times \sigma^2_{Min} + 2 \times (.31)(.69)\sigma_{Max,Min}} =$ $\sqrt{.31^2 \times .1764 + .69^2 \times .0784} = 23.30\%$.

B. Which has the highest Sharpe ratio, *Max*, *Min* or the minimum variance portfolio combination of *Max* and *Min*?

SOLUTION: The Sharpe ratio is computed as the excess return on the security divided by its standard deviation. Therefore,

Sharpe Ratio$_{Max}$ = (.20-.04)/.42 = 38.10%;

Sharpe Ratio$_{Min}$ = (.12-.04)/.28 = 28.53%; and

Sharpe Ratio$_{MVP}$ = (.145-.04)/.233 = 44.90%.

Therefore, the minimum variance portfolio (MVP) combination of *Max* and *Min* has the highest Sharpe ratio (44.90%), while *Max* has the second highest at 38.10%. The Sharpe ratio for *Min* is significantly lower (only 28.53%).

C. Suppose the correlation between *Max* and *Min* is -1. If this were the case, there would be an arbitrage opportunity, since a combination of *Max* and *Min* exists that is riskless and yields a higher expected return than the riskless asset. Describe the characteristics of a portfolio strategy that would enable you to generate positive profits without having to bear any risk or put up any of your own money. Assume that there are no restrictions on short sales or margin requirements.

SOLUTION: $w_{Min} = \dfrac{.1764 - (-1)(.28)(.42)}{.0784 + .1764 - (-2)(.28)(.42)}$ =.294/.49 = 3/5. The expected return for this portfolio is $E(r_p) = \dfrac{3}{5} \times E(r_{Max}) + \dfrac{2}{5} \times E(r_{Min}) = .60(20\%) + .40(12\%)$ $= 15.20\%$, and $\sigma_p = 0$ because $\rho_{Min,Max} = -1$. We can generate positive profits without having to bear any risk or put up any of our own money by simply choosing the following set of weights: $w_{Min} = 3/5$, $w_{Max} = 2/5$, and $w_{r_f} = -1$. In other words, we go long 100 percent in the riskless combination of *Max* and *Min*, and 100 percent short in the riskless asset; i.e., we fund our investment in the combination of *Max* and *Min* by borrowing an equivalent sum of money at the riskless rate of interest.

D. Now suppose the expected return to the market portfolio is 14%, and the standard deviation of the market portfolio is 25 %. Assuming that the CAPM holds, what are the betas for *Max* and *Min*?

SOLUTION: According to the CAPM, $E(r_{Max}) = r_f + [E(r_m) - r_f]\,\beta_{Max}$; therefore, $\beta_{Max} = \left(\frac{[E(r_{Max}) - r_f]}{[E(r_m) - r_f]}\right)$ = (.20-.04)/.10 = 1.6. Similarly we can find β_{Min} by calculating the ratio $\left(\frac{[E(r_{Min}) - r_f]}{[E(r_m) - r_f]}\right)$ =(.12-.04)/.10 = .8.